

## Παραγοντοποίηση - Ασκήσεις

$$\boxed{1} \text{ i) } 4x^2 - 6x = 2x(2x - 3)$$

$$\text{ii) } 3x + 3 = 3(x + 1)$$

$$\text{iii) } x^4 - x^2y = x^2(x^2 - y)$$

$$\text{iv) } 5a^2xy - 10a^5x^3 = 5a^2x(y - 2a^3x^2)$$

$$\boxed{2} \text{ i) } 3x(x - y) + 5y(x - y) = (x - y)(3x + 5y)$$

$$\text{ii) } 5(x + y) + a(x + y) = (x + y)(5 + a)$$

$$\begin{aligned} \text{iii) } x(a - \beta) - 3(\beta - a) &= x(a - \beta) + 3(-\beta + a) \\ &= x(a - \beta) + 3(a - \beta) \\ &= (a - \beta)(x + 3) \end{aligned}$$

$$\begin{aligned} \text{iv) } 7w(a + \beta) - a - \beta &= 7w(a + \beta) - (a + \beta) \\ &= (a + \beta)(7w - 1) \end{aligned}$$

$$\begin{aligned} \text{v) } 3x(x - 1) - x + 1 &= 3x(x - 1) - (x - 1) \\ &= (x - 1)(3x - 1) \end{aligned}$$

$$\boxed{3} \text{ i) } 3x^3 - x^2y + 6x - 2y = x^2(3x-y) + 2(3x-y) = \\ = (3x-y)(x^2+2)$$

$$\text{ii) } 10x^3 + 4xy^2 - 6y^3 - 15x^2y = 2x(5x^2+2y^2) - 3y(2y^2+5x^2) \\ = 2x(5x^2+2y^2) - 3y(5x^2+2y^2) \\ = (5x^2+2y^2)(2x-3y)$$

$$\text{iii) } xy + x^2 - x - y = x(y+x) - (x+y) = \\ x(x+y) - (x+y) = \\ (x+y)(x-1)$$

$$\text{iv) } a - ax + \beta - \beta x = a(1-x) + \beta(1-x) \\ = (1-x)(a+\beta)$$

$$\text{v) } xy^2 + xw - y^2z - wz = x(y^2+w) - z(y^2+w) \\ = (y^2+w)(x-z)$$

$$\boxed{4} \text{ i) } x^2(a+b) + x(a+b) = x(a+b)(x+1)$$

$$\text{ii) } 2x(a-k) + k - a = 2x(a-k) - a + k \\ = 2x(a-k) - (a-k) \\ = (a-k)(2x-1)$$

$$\text{iii) } 36x^2y^2 - 16 = (6xy)^2 - 4^2 = (6xy-4)(6xy+4)$$

$$\text{iv) } a^3 - a^2b + ab^2 - b^3 = a^2(a-b) + b^2(a-b) \\ = (a-b)(a^2+b^2)$$

$$\begin{aligned}
 \text{v)} \quad x(x+2) - y(y+2) &= x^2 + 2x - y^2 - 2y \\
 &= x^2 - y^2 + 2x - 2y \\
 &= (x-y)(x+y) + 2(x-y) \\
 &= (x-y)(x+y+2)
 \end{aligned}$$

$$\begin{aligned}
 \text{vi)} \quad x^4 + x^3y - xy^3 - y^4 &= x^3(x+y) - y^3(x+y) \\
 &= (x+y)(x^3 - y^3) \\
 &= (x+y)(x-y)(x^2 + xy + y^2)
 \end{aligned}
 \left. \vphantom{\begin{aligned} x^4 + x^3y - xy^3 - y^4 \\ = x^3(x+y) - y^3(x+y) \\ = (x+y)(x^3 - y^3) \\ = (x+y)(x-y)(x^2 + xy + y^2) \end{aligned}} \right\} \begin{array}{l} \text{Ettos} \\ \text{ans!} \end{array}$$

$$\begin{aligned}
 \text{vii)} \quad (x+y)^4 - (x+y)^2 &= (x+y)^2 [(x+y)^2 - 1] \\
 &= (x+y)^2 (x+y-1)(x+y+1)
 \end{aligned}$$

$$\begin{aligned}
 \text{viii)} \quad \underbrace{a^2 - b^2} + a - b &= (a-b)(a+b) + (a-b) \\
 &= (a-b)(a+b+1)
 \end{aligned}$$

$$\boxed{15} \text{ i)} (x+1)(x+3) = 0 \quad \text{'Apa: } \begin{array}{l} x+1=0 \\ x=-1 \end{array} \quad \begin{array}{l} | \\ \eta \end{array} \quad \begin{array}{l} x+3=0 \\ x=-3 \end{array}$$

$$\text{ii)} (x-1)(2x+7) = 0 \quad \begin{array}{l} x-1=0 \\ x=1 \end{array} \quad \begin{array}{l} | \\ \eta \end{array} \quad \begin{array}{l} 2x+7=0 \\ 2x=-7 \\ x=-\frac{7}{2} \end{array}$$

$$\begin{aligned}
 \text{iii)} \quad x^2 - 121 &= 0 \\
 x^2 - 11^2 &= 0 \\
 (x-11)(x+11) &= 0
 \end{aligned}
 \left. \vphantom{\begin{aligned} x^2 - 121 = 0 \\ x^2 - 11^2 = 0 \\ (x-11)(x+11) = 0 \end{aligned}} \right\} \begin{array}{l} x-11=0 \\ x=11 \end{array} \quad \begin{array}{l} | \\ \eta \end{array} \quad \begin{array}{l} x+11=0 \\ x=-11 \end{array}$$

$$\begin{aligned}
 \text{iv)} \quad 8x^2 + 24x &= 0 \\
 8x(x+3) &= 0
 \end{aligned}
 \left. \vphantom{\begin{aligned} 8x^2 + 24x = 0 \\ 8x(x+3) = 0 \end{aligned}} \right\} \begin{array}{l} x=0 \\ \eta \end{array} \quad \begin{array}{l} x+3=0 \\ x=-3 \end{array}$$

$$6 \text{ a) } \frac{x+5}{3}, x \in \mathbb{R}$$

(x οποιαδήποτε πραγματικός αριθμός)

$$\beta) \frac{x+2}{x-3}, x-3 \neq 0, x \neq 3$$

$$\gamma) \frac{x^2+5}{2x+8}, 2x+8 \neq 0$$

$$2x \neq -8$$

$$x \neq -4$$

$$\delta) \frac{3+x}{x^2-4} = \frac{3+x}{(x+2)(x-2)}$$

Άρα:  $x+2 \neq 0, x \neq -2$

ΚΑΙ  $x-2 \neq 0, x \neq 2$

$$\epsilon) \frac{5x}{x^2+2}, x \in \mathbb{R}$$

(οποιαδήποτε πραγματικός αριθμός, αφού πάντα  $x^2 \geq 0$ )

$$\sigma\tau) \frac{x+1821}{x^3-9x} = \frac{x+1821}{x(x^2-9)} = \frac{x+1821}{x(x-3)(x+3)}$$

Άρα:  $x \neq 0$

ΚΑΙ  $x-3 \neq 0, x \neq 3$

ΚΑΙ  $x+3 \neq 0, x \neq -3$

$$7) \frac{x^2-1}{x+1} = \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}} = x-1$$

$$\frac{x^2+x}{2x+2} = \frac{x\cancel{(x+1)}}{2\cancel{(x+1)}} = \frac{x}{2}$$

$$\frac{4x^2-xy}{12xy-3y^2} = \frac{x\cancel{(4x-y)}}{3y\cancel{(4x-y)}} = \frac{x}{3y}$$

$$\frac{x^3-x}{x^2+x} = \frac{x(x^2-1)}{x(x+1)} = \frac{\cancel{x}(x-1)\cancel{(x+1)}}{\cancel{x}(x+1)} = x-1$$

$$\frac{2x-3}{3-2x} = \frac{2x-3}{-(-3+2x)} = \frac{\cancel{2x-3}}{\cancel{-(2x-3)}} = -1$$

$$\frac{a\beta}{a^2\beta^2 - a\beta} = \frac{\cancel{a\beta}}{\cancel{a\beta}(a\beta-1)} = \frac{1}{a\beta-1}$$

$$\frac{x^4 - y^4}{x^2 - y^2} = \frac{(x^2)^2 - (y^2)^2}{x^2 - y^2} = \frac{(x^2 + y^2)\cancel{(x^2 - y^2)}}{\cancel{x^2 - y^2}} = x^2 + y^2$$

$$\frac{x^2 + xy}{y^2 + xy} = \frac{x(x+y)}{y(y+x)} = \frac{\cancel{x(x+y)}}{\cancel{y(x+y)}} = \frac{x}{y}$$

$$\frac{x^2 + 2x - 3}{x^2 - 3x + 2}$$

$$\begin{aligned} * x^2 + 2x + 3 &= x^2 + 3x - x - 3 \\ &= x(x+3) - (x+3) \\ &= (x+3)(x-1) \end{aligned}$$

$$\begin{aligned} ** x^2 - 3x + 2 &= x^2 - x - 2x + 2 \\ &= x(x-1) - 2(x-1) \\ &= (x-1)(x-2) \end{aligned}$$

$$* \frac{(x+3)\cancel{(x-1)}}{\cancel{(x-1)}(x-2)}$$

$$** \frac{\cancel{(x-1)}(x-2)}{\cancel{(x-1)}(x-2)}$$

$$= \frac{x+3}{x-2}$$

$$x-2$$